$$|\dot{\alpha}|_{\max} = \frac{b_2}{a^2(1-c)^2} \tag{51}$$

These occur at the end points of the trajectory ($p = 0$ and $p = 2\pi$).

Once the end-effector trajectory is defined in this manner by Eqs. (39) and (40), it is a straightforward task to express the vectors $\boldsymbol{y}$, $\boldsymbol{v}$, and $\boldsymbol{a}$ that are needed in Eqs. (23-25), (29), and (33), for example. Specifically, if the unit vector along the end-effector path is $\boldsymbol{u}$ and the unit vector representing the fixed axis about which the rotation θ is made is $\boldsymbol{w}$, both expressed in the base frame, then

$$\boldsymbol{y} = \begin{bmatrix} y & \boldsymbol{u} \\ \theta & \boldsymbol{w} \end{bmatrix} \tag{52}$$

$$\boldsymbol{v} = \begin{bmatrix} v & \boldsymbol{u} \\ \omega & \boldsymbol{w} \end{bmatrix} \tag{53}$$

$$\boldsymbol{a} = \begin{bmatrix} a & \boldsymbol{u} \\ \alpha & \boldsymbol{w} \end{bmatrix} \tag{54}$$

in which y, θ, v, ω, a, and α are as expressed by Eqs. (39), (40), (35), (36), (43), and (44).

Conclusions

Design of the end-effector trajectory of a robotic manipulator according to motion specifications such as acceleration and jerk limits, and the design of the joint trajectories to minimize base reactions have been developed in this paper. The techniques presented have potential applications in space manipulators. Curtate cycloids were used to represent the end-effector motion. These have the benefits of finite acceleration and finite jerk. Kinematic redundancy is exploited in designing joint trajectories that will minimize a quadratic cost function in base reaction components. An application of the technique is given in Ref. 1.

The question of whether the corresponding results are global will be addressed in future work. Also, the effect of using alternative cost functions, such as a quadratic integral function, needs to be studied. In this paper, we have not addressed the problem of whether the actuators are capable of generating the torques required by the trajectory.[8] The sensitivity of the weighting matrix in emphasizing certain reaction components is being investigated.[8] In the example given in this paper, polynomial shape functions were used to represent redundant-joint trajectories. The potential advantages and possible disadvantages of using other types of shape functions should be studied as well.

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Jacobi Method for Unsymmetric Eigenproblems

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Introduction

THE complete solution of the eigenvalue problem via the Jacobi method is widely used in symmetric problems. Moreover, where the problem can be stated as the product of two symmetric matrices, as in problems of dynamics, a solution consisting of two Jacobi solutions is common practice. This paper offers a complete solution of the unsymmetric problem via a variation of the Jacobi method. The eigenvalues emerge in algebraic order, and there is no restriction of symmetry in the problem. Proof of convergence is included.

In the matric expression

$$AV = VD \tag{1}$$

V and D are the matrices of eigenvectors and eigenvalues of the nondegenerate matrix A. Of course, D is a diagonal matrix.

Matrices V and D may be found by way of matrices X, an orthogonal matrix, and Λ, a triangular matrix that contains the eigenvalues of A on its diagonal, such that

$$AX = X\Lambda \tag{2}$$

Let Y be the matrix of eigenvectors of Λ. Then,

$$\Lambda Y = YD \tag{3}$$

or

$$\Lambda = YDY^{-1} \tag{4}$$

Equations (2) and (4) may be combined to get

$$AX = XYDY^{-1} \tag{5}$$

or

$$AXY = XYD \tag{6}$$

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It follows from Eq. (1) and (6) that

$$V = XY$$

Note that given Λ, the computation of Y is simple and well documented. (See, for example, Ref. 1, p. 56).

Problem

Given A, a nondegenerate real matrix, find the orthogonal matrix X and the triangular matrix Λ such that

$$AX = X\Lambda \tag{7}$$

or

$$X^T AX = \Lambda \tag{8}$$

Solution

Let X be the product of F similarity transformations X_i, or

$$X = X_1 X_2 \cdots X_F = \prod_{i=1}^{F} X_i \tag{9}$$

where F is just large enough to obtain a converged solution for X.

Equation (8) may then be written as

$$X_F^T \cdots X_2^T X_1^T A X_1 X_2 \cdots X_F = \Lambda \tag{10}$$

Following a finite number of transformations,

$$A_K = X_K^T A_{K-1} X_K$$

will lead to the desired lower triangular matrix Λ if each X annihilated an element a_{ij}, $j > i$, while leaving unchanged the sum of the squares of the elements that are altered. Transformation matrix X can be tailored to fulfill these requirements.

Let (see Ref. 2, p. 87)

$$X_K = \begin{array}{cc} \begin{array}{ccccccccccc} &&&i&&&&j&&& \end{array} & \\ \left[\begin{array}{ccccccccccc} 1&&&&&&&&&& \\ &\ddots&&&&&&&&& \\ &&1&&&&&&&& \\ &&&S&&&&C&&& \\ &&&&1&&&&&& \\ &&&&&\ddots&&&&& \\ &&&&&&1&&&& \\ &&&C&&&&-S&&& \\ &&&&&&&&1&& \\ &&&&&&&&&\ddots& \\ &&&&&&&&&&1 \end{array}\right] & \begin{array}{c} \\ \\ \\ i \\ \\ \\ \\ j \\ \\ \\ \\ \end{array} \end{array} = X_K^T = X_K^{-1} \tag{11}$$

where C and S are to be determined by the condition that $a_{ij}^K = 0$ and X_K be orthogonal.

Focusing attention on rows and columns i and j, these quantities may be solved as follows:

$$A_K = \begin{bmatrix} S & C \\ C & -S \end{bmatrix}_K \begin{bmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{bmatrix}_{K-1} \begin{bmatrix} S & C \\ C & -S \end{bmatrix}_K \tag{12}$$

$$= \left[\begin{array}{c|c} Sa_{ii} + Ca_{ji} & Sa_{ij} + Ca_{jj} \\ Ca_{ii} - Sa_{ji} & Ca_{ij} - Sa_{jj} \end{array}\right] \begin{bmatrix} S & C \\ C & -S \end{bmatrix} \tag{13}$$

$$= \begin{bmatrix} (Sa_{ii} + Ca_{ji})S + (Sa_{ij} + Ca_{jj})C & (Sa_{ii} + Ca_{ji})C - (Sa_{ij} + Ca_{jj})S \\ (Ca_{ii} - Sa_{ji})S + (Ca_{ij} - Sa_{jj})C & (Ca_{ii} - Sa_{ji})C - (Ca_{ij} - Sa_{jj})S \end{bmatrix} \tag{14}$$

$$a_{ij}^K = 0 = SCa_{ii} + C^2 a_{ji} - S^2 a_{ij} - SCa_{jj} \tag{15}$$

$$= C^2 a_{ji} + (a_{ii} - a_{jj})SC - S^2 a_{ij} \tag{16}$$

$$= a_{ji} + (a_{ii} - a_{jj}) \frac{S}{C} - \left(\frac{S}{C}\right)^2 a_{ij} \tag{17}$$

```
                              MATRIX A, THE PROBLEM MATRIX

   2.000000D+00   5.000000D+00   5.000000D+00
   4.000000D+00   3.000000D+00   5.000000D+00
   4.000000D+00   4.000000D+00   4.000000D+00

ROT'N NO. = 1, I = 1, J = 2, S = 7.071068D-01, C = -7.071068D-01, A (I, J) = 0.0
ROT'N NO. = 2, I = 2, J = 3, S = 7.492686D-01, C = -6.622662D-01, A (I, J) = 1.332268D-15

   2 ORTHOGONAL ROTATIONS TO REACH LARGEST COMPUTED ZERO OF 1.3323D-15

       MATRIX LAMDA = MATRIX A TRIANGULARIZED, WITH THE EIGENVALUES ON THE DIAGONAL

              -2.000000D+00            0.0                     0.0
              -5.351919D-02            1.200000D+01           -1.332268D-15
               5.865786D-01           -1.087857D-01           -1.000000D+00

                          MATRIX V = EIGENVECTORS OF MATRIX A

               1.000000D+00            1.000000D+00           -6.250000D-01
              -4.000000D-01            1.000000D+00           -6.250000D-01
              -4.000000D-01            1.000000D+00            1.000000D+00

                          BACK-SUBSTITUTION VDV-1  ?= A

               2.000000D+00            5.000000D+00            5.000000D+00
               4.000000D+00            3.000000D+00            5.000000D+00
               4.000000D+00            4.000000D+00            4.000000D+00
```

Fig. 1 Real problem with real solution.

Then,

$$\frac{S}{C} = \frac{(a_{ii} - a_{jj}) \pm \sqrt{(a_{ii} - a_{jj})^2 + 4a_{ij}a_{ji}}}{2a_{ij}} \tag{18}$$

Because of the orthogonality of X_K

$$S^2 + C^2 = 1 \tag{19}$$

$$(S^2/C^2) + 1 = (1/C^2) \tag{20}$$

$$C^2 = \frac{1}{1 + (S/C)^2} \tag{21}$$

$$C = \frac{1}{\pm\sqrt{1 + (S/C)^2}} \tag{22}$$

Then,

$$S = C(S/C) \tag{23}$$

Choosing the sign of the radical in Eq. (22) is of no consequence whatever. However, in Eq. (18) the choice of sign on the radical determines the sign of the quantity $(a_{ii} - a_{jj})_K$.

Proof:

$$\begin{aligned}(a_{ii} - a_{jj})_K &= a_{ii}S^2 + (a_{ij} + a_{ji})SC \\ &\quad + a_{jj}C^2 - a_{jj}S^2 + (a_{ij} + a_{ji})SC - a_{ii}C^2 \\ &= (a_{ii} - a_{jj})(S^2 - C^2) + 2(a_{ij} + a_{ji})SC\end{aligned}$$

From Eq. (15):

$$SC(a_{ii} - a_{jj}) = S^2 a_{ij} - C^2 a_{ji}$$

$$a_{ii} - a_{jj} = \frac{S}{C} a_{ij} - \frac{C}{S} a_{ji}$$

Then,

$$\begin{aligned}(a_{ii} - a_{jj})_K &= \left(\frac{S}{C} a_{ij} - \frac{C}{S} a_{ji}\right)(S^2 - C^2) + 2(a_{ij} + a_{ji})SC \\ &= S^2 \frac{S}{C} a_{ij} - SCa_{ji} - SCa_{ij} \\ &\quad + C^2 \frac{C}{S} a_{ji} + 2a_{ij}SC + 2a_{ji}SC \\ &= S^2 \frac{S}{C} a_{ij} + SCa_{ji} + SCa_{ij} + C^2 \frac{C}{S} a_{ji} \\ &= S^2 \frac{S}{C} a_{ij} + S^2 \frac{C}{S} a_{ji} + C^2 \frac{S}{C} a_{ij} + C^2 \frac{C}{S} a_{ji} \\ &= (S^2 + C^2) \frac{S}{C} a_{ij} + (S^2 + C^2) \frac{C}{S} a_{ji} \\ &= \frac{S}{C} a_{ij} + \frac{C}{S} a_{ji}\end{aligned} \tag{24}$$

Substituting Eq. (18) into Eq. (24), one obtains

$$(a_{ii} - a_{jj})_K = \frac{(a_{ii} - a_{jj}) + \sqrt{(a_{ii} - a_{jj})^2 + 4a_{ij}a_{ji}}}{2} + \frac{2a_{ij}a_{ji}}{(a_{ii} - a_{jj}) + \sqrt{(a_{ii} - a_{jj})^2 + 4a_{ij}a_{ji}}} \tag{25}$$

```
                                        MATRIX A, THE PROBLEM MATRIX

     2.000000D+00              -5.000000D+00              -5.000000D+00
     4.000000D+00               3.000000D+00              -5.000000D+00
     4.000000D+00               4.000000D+00               4.000000D+00

ROT'N NO. =  1, I = 1, J = 2, S = 6.666667D-01, C =  ( 8.333333D-02,  7.406829D-01I), A (I, J) = (-2.220446D-16,  2.220446D-16I)
ROT'N NO. =  2, I = 1, J = 3, S = 8.169792D-01, C =  (-3.420714D-01,  4.642544D-01I), A (I, J) = (-4.440892D-16,  1.249001D-15I)
ROT'N NO. =  3, I = 1, J = 2, S = 9.844688D-01, C =  (-1.701952D-01,  4.306609D-02I), A (I, J) = ( 1.804112D-16,  2.220446D-16I)
ROT'N NO. =  4, I = 2, J = 3, S = 7.434524D-01, C =  (-5.233440D-01,  4.164007D-01I), A (I, J) = ( 8.881784D-16, -4.440892D-16I)
ROT'N NO. =  5, I = 1, J = 3, S = 9.973119D-01, C =  (-3.045939D-02,  6.664220D-02I), A (I, J) = ( 1.526557D-16,  4.943962D-17I)
ROT'N NO. =  6, I = 2, J = 3, S = 9.998729D-01, C =  ( 1.295444D-02,  9.295895D-03I), A (I, J) = ( 1.387779D-17,  2.775558D-17I)
ROT'N NO. =  7, I = 1, J = 2, S = 9.992671D-01, C =  (-3.335155D-02,  1.878567D-02I), A (I, J) = ( 0.0         ,  4.163336D-17I)
ROT'N NO. =  8, I = 1, J = 3, S = 9.999993D-01, C =  (-1.218674D-03, -5.756743D-05I), A (I, J) = (-8.131416D-19, -1.734723D-18I)
ROT'N NO. =  9, I = 1, J = 2, S = 1.261913D-01, C =  ( 1.508230D-01, -9.804735D-01I), A (I, J) = ( 3.457651D-13,  1.736972D-12I)
ROT'N NO. = 10, I = 1, J = 3, S = 1.000000D+00, C =  (-2.706028D-04, -6.547042D-05I), A (I, J) = ( 1.084202D-19,  7.047314D-19I)
ROT'N NO. = 11, I = 1, J = 2, S = 1.261961D-01, C =  ( 1.500615D-01, -9.805897D-01I), A (I, J) = ( 3.910205D-13,  5.178857D-12I)
ROT'N NO. = 12, I = 1, J = 3, S = 1.000000D+00, C =  (-3.414702D-05, -8.290896D-06I), A (I, J) = ( 2.032879D-20,  0.0         I)
ROT'N NO. = 13, I = 1, J = 2, S = 1.261978D-01, C =  ( 1.502372D-02, -9.805625D-01I), A (I, J) = ( 1.271205D-13,  3.476401D-11I)
ROT'N NO. = 14, I = 1, J = 3, S = 1.000000D+00, C =  (-4.309633D-06, -1.045551D-06I), A (I, J) = ( 0.0         ,  0.0         I)

      14 UNITARY ROTATIONS TO REACH LARGEST COMPUTED ZERO OF 5.9069D-06

                         MATRIX LAMBDA = MATRIX A TRIANGULARIZED, WITH THE EIGENVALUES ON THE DIAGONAL

( 2.830836D+00,  7.686732D+00I)          (-1.270882D-06, -5.452663D-06I)          ( 0.0         ,  0.0         I)
(-1.926355D-02,  1.257475D-01I)          ( 2.830837D+00, -7.686732D+00I)          ( 4.295847D-06, -6.578160D-07I)
( 3.942156D-03, -1.651640D+01I)          ( 1.634301D-01,  1.227195D-02I)          ( 3.338327D+00, -5.478801D-08I)

                                    MATRIX V = EIGENVECTORS OF MATRIX A

( 1.000000D+00,  1.387779D-17I)          ( 1.000000D+00,  1.387779D-17I)          (-8.732454D-01,  3.297629D-07I)
( 3.975763D-01, -8.958258D-01I)          ( 3.975758D-01,  8.958260D-01I)          ( 1.000000D+00,  0.0         I)
(-5.637435D-01, -6.415206D-01I)          (-5.637442D-01,  6.415207D-01I)          (-7.662623D-01,  7.209366D-07I)

                                      BACK-SUBSTITUTION VDV^-1  ?=  A

( 1.999998D+00, -7.748659D-07I)          (-5.000000D+00,  3.262581D-06I)          (-4.999999D+00, -2.017952D-07I)
( 4.000001D+00,  1.043387D-06I)          ( 3.000000D+00,  2.181205D-06I)          (-4.999997D+00, -1.253839D-06I)
( 4.000003D+00,  2.244706D-06I)          ( 4.000000D+00, -8.715412D-07I)          ( 4.000002D+00, -1.406339D-06I)
```

Fig. 2 Real problem with complex solution.

which reduces to

$$(a_{ii} - a_{jj})_K = \sqrt{(a_{ii} - a_{jj})^2 + 4a_{ij}a_{ji}}$$

Thus, after the annihilation of the off-diagonal element a_{ij}, $j > i$, the eigenvalues on the diagonal will be in descending order when $\sqrt{(a_{ii} - a_{jj})^2 + 4a_{ij}a_{ji}}$ is assigned a positive sign, and they will be in ascending order with a negative sign.

The sum of the squares of the elements in the rows that have been altered during the operation $X_K^T A_{K-1}$ remains unchanged. The proof is that for each column k of the affected rows i and j

$$\begin{aligned}(a_{ik}^2 + a_{jk}^2)_K &= (Sa_{ik} + Ca_{jk})_{K-1}^2 + (Ca_{ik} - Sa_{jk})_{K-1}^2 \\ &= S^2(a_{ik}^2 + a_{jk}^2) + C^2(a_{ik}^2 + a_{jk}^2) \\ &= (S^2 + C^2)(a_{ik}^2 + a_{jk}^2) \\ &= (a_{ik}^2 + a_{jk}^2)_{K-1}\end{aligned}$$

The preceding procedure can be repeated to show the invariance in the sum of the squares of the elements in the columns transformed from $X_K^T A_{K-1}$ to $X_K^T A_{K-1} X_K$.

It can be shown that the preceding discussion also applies to problems whose solutions are complex when the variable C in matrix X_K of Eq. (11), now a unitary matrix, is replaced by its complex counterpart above the diagonal, and its complex conjugate below the diagonal. But since the motion of algebraic order is nonexistent with complex numbers, arbitrating the sign of the quantity $(a_{ii} - a_{jj})_K$ via Eq. (18) serves no purpose. This lack of order seems to be the reason why some problems whose solution is complex will fail to converge—without deverging.

Numerical Examples

Figure 1 shows the details of the solution of a real problem with a real solution. Note that although the sign of the radical in Eq. (18) is set to produce eigenvalues of descending order on the diagonal of matrix Λ, the second and third eigenvalues do not comply to this expectation. This is because all above-diagonal elements had already been sufficiently annihilated just before these two values were to be processed.

Figure 2 shows the details of the solution of a real problem with a complex solution.

Conclusion

An eigenvalue problem solution using the Jacobi method was derived. In this method, the eigenvalues are inherently generated in algebraic order. This order is a necessary ingredient of the process since its absence will impede convergence of the unsymmetric problem solution. The notion of algebraic order is nonexistent with complex numbers. For that reason, this method will not always be convergent with problems whose solution is complex.

Acknowledgments

Dividing Eq. (16) by C^2 to arrive at Eq. (17), the prime mover behind this method, was the suggestion of J. Pickard. Also, thanks to C. G. Dietz, principal engineer at McDonnell Douglas Space Systems Company, Huntington Beach, CA, for the proof following Eq. (23).

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Generalized Proportional-Plus-Derivative Compensators for a Class of Uncertain Plants

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Introduction

THIS paper considers controller design for a class of systems consisting of a positive-real transfer function $P(s)$ followed by an integrator (Fig. 1). Examples of such plants include flexible space structures, which must be controlled with specified precision in attitude and shape. We assume that the order of $P(s)$, as well as the values of its parameters, are not accurately known; the only information available with certainty is that $P(s)$ is positive real, as defined below.[1]

Definition. A rational function $G(s)$ of the complex variable s is said to be positive real (PR) if 1) $G(s)$ is real when s is real, and 2) $\mathrm{Re}[G(s)] \geq 0$ for all $\mathrm{Re}[s] \geq 0$, where Re[.] denotes the real part.

It can be shown that PR functions have no zeros or poles in the open right half of the complex plane, and that the poles on the imaginary axis are simple (i.e., of multiplicity one), with non-negative residues.[1] It can be also shown that PR functions must have relative degree between -1 and 1 (relative degree is the difference between the degrees of the denominator and the numerator polynomials).

An example of the type of system shown in Fig. 1 arises in the study of large, flexible space structures. For example, the linearized single-axis rotational motion of a flexible space structure can be described by the transfer function:

$$G(s) = \frac{Y_a(s)}{U(s)} = \frac{1}{Js^2} + \sum_{i=1}^{n_q} \frac{\phi_i \psi_i}{s^2 + 2\rho_i \omega_i s + \omega_i^2} \tag{1}$$

where J is the moment of inertia, $y_a(t)$ denotes the total "attitude" at the sensor location, $u(t)$ is the applied control torque, ω_i, ρ_i, ϕ_i, and ψ_i are the elastic mode frequency, inherent damping ratio, and rotational component of the mode shape at the actuator and sensor locations, respectively, for the ith elastic mode. (ω_i are positive, and ρ_i are small positive numbers on the order of 0.01.) The number of modes included in the model is n_q. In theory, there are infinite modes; therefore, unmodeled dynamics are always present. If the actuator and sensor are collocated, then $\phi_i = \psi_i$. Obviously, $G(s)$ in Eq. (1) is not positive real because its relative degree is 2. However, consider the transfer function $P(s)$ between $u(t)$ and $y_r(t)$, where $y_r(t)$ is attitude rate $[=\dot{y}_a(t)]$:

$$P(s) = sG(s) = \frac{1}{Js} + \sum_{i=1}^{n_q} \frac{\phi_i^2 s}{s^2 + 2\rho_i \omega_i s + \omega_i^2} \tag{2}$$

It can be seen that $P(s)$ in Eq. (2) is indeed positive real. Furthermore, it is clear that $P(s)$ has only one pole on the imaginary axis ($s = 0$) and no zeros on the imaginary axis.

Definitions and Preliminaries

A system represented by the rational transfer function $H(s)$ (wherein the numerator and denominator polynomials are

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